

# On $\mathcal{N} = 2$ supergravity and projective superspace: Dual formulations

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*Dedicated to Professor I. L. Buchbinder  
On the Occasion of His 60th Birthday*

## Abstract

The superspace formulation for four-dimensional  $\mathcal{N} = 2$  matter-coupled supergravity recently developed in [1] makes use of a new type of conformal compensator with infinitely many off-shell degrees of freedom: the so-called covariant weight-one polar hypermultiplet. In the present note we prove the duality of this formulation to the known minimal  $(40 + 40)$  off-shell realization for  $\mathcal{N} = 2$  Poincaré supergravity involving the improved tensor compensator. Within the latter formulation, we present new off-shell matter couplings realized in terms of covariant weight-zero polar hypermultiplets. We also elaborate upon the projective superspace description of vector multiplets in  $\mathcal{N} = 2$  conformal supergravity. An alternative superspace representation for locally supersymmetric chiral actions is given. We present a model for massive improved tensor multiplet with both “electric” and “magnetic” types of mass terms.

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# 1 Introduction

Recently, we have developed the superspace formulation for four-dimensional  $\mathcal{N} = 2$  matter-coupled supergravity [1], extending the earlier construction for 5D  $\mathcal{N} = 1$  supergravity [2, 3]. From the purely geometrical point of view, this approach makes use of Grimm's curved superspace geometry [4], which is perfectly suitable to describe  $\mathcal{N} = 2$  conformal supergravity and has a simple relation to Howe's superspace formulation [5]. Kinematically, matter fields in [1] are described in terms of covariant projective supermultiplets which are curved-space versions of the superconformal projective multiplets [6] living in rigid projective superspace [7, 8]. In addition to the local  $\mathcal{N} = 2$  superspace coordinates  $z^M = (x^m, \theta_i^\mu, \bar{\theta}_{\dot{\mu}}^i)$ , where  $m = 0, 1, \dots, 3$ ,  $\mu = 1, 2$ ,  $\dot{\mu} = 1, 2$  and  $i = \underline{1}, \underline{2}$ , such a supermultiplet,  $Q^{(n)}(z, u^+)$ , depends on auxiliary isotwistor variables  $u_i^+ \in \mathbb{C}^2 \setminus \{0\}$ , with respect to which  $Q^{(n)}$  is holomorphic and homogeneous,  $Q^{(n)}(c u^+) = c^n Q^{(n)}(u^+)$ , on an open domain of  $\mathbb{C}^2 \setminus \{0\}$  (the integer parameter  $n$  is called the weight of  $Q^{(n)}$ ). In other words, such superfields are intrinsically defined in  $\mathbb{CP}^1$ . The covariant projective supermultiplets are required to be annihilated by half of the supercharges<sup>1</sup>

$$\mathcal{D}_\alpha^+ Q^{(n)} = \bar{\mathcal{D}}_{\dot{\alpha}}^+ Q^{(n)} = 0, \quad \mathcal{D}_\alpha^+ := u_i^+ \mathcal{D}_\alpha^i, \quad \bar{\mathcal{D}}_{\dot{\alpha}}^+ := u_i^+ \bar{\mathcal{D}}_{\dot{\alpha}}^i, \quad (1.1)$$

with  $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha^i, \bar{\mathcal{D}}_{\dot{\alpha}}^i)$  the covariant superspace derivatives.

In the approach of [1], the dynamics of supergravity-matter systems is described by a locally supersymmetric action of the form:

$$S = \frac{1}{2\pi} \oint (u^+ du^+) \int d^4x d^4\theta d^4\bar{\theta} E \frac{\mathbf{W} \bar{\mathbf{W}} \mathcal{L}^{++}}{(\Sigma^{++})^2}, \quad E^{-1} = \text{Ber}(E_A^M), \quad (1.2)$$

where

$$\Sigma^{++} := \frac{1}{4} \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \mathbf{W} = \frac{1}{4} \left( (\bar{\mathcal{D}}^+)^2 + 4\tilde{S}^{++} \right) \bar{\mathbf{W}} = \Sigma^{ij} u_i^+ u_j^+. \quad (1.3)$$

Here the Lagrangian  $\mathcal{L}^{++}(z, u^+)$  is a covariant real projective multiplet of weight two,  $\mathbf{W}(z)$  is the covariantly chiral field strength of an Abelian vector multiplet (i.e. the first superconformal compensator), such that the body of  $\mathbf{W}(z)$  is everywhere non-vanishing,  $S^{++}(z, u^+) = S^{ij}(z) u_i^+ u_j^+$  and  $\tilde{S}^{++}(z, u^+) = \bar{S}^{ij}(z) u_i^+ u_j^+$  are special dimension-1 components of the supertorsion; see [1] for more detail. The action (1.2) can be shown to be invariant under the supergravity gauge transformations, and is also manifestly super-Weyl invariant. It can also be rewritten in the equivalent form

$$S = \frac{1}{2\pi} \oint (u^+ du^+) \int d^4x d^4\theta d^4\bar{\theta} E \frac{\mathcal{L}^{++}}{S^{++} \tilde{S}^{++}} \quad (1.4)$$

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<sup>1</sup>In the rigid supersymmetric case, such constraints in isotwistor superspace  $\mathbb{R}^{4|8} \times \mathbb{CP}^1$  were introduced first by Rosly [9], and later by the harmonic [10, 11] and projective [7, 8] superspace practitioners.

in which, however, the super-Weyl invariance is not manifest.

In [1], we presented a family of supergravity-matter systems in which the matter hypermultiplets are described by covariant weight-zero polar multiplets<sup>2</sup>, and the second superconformal compensator is identified with a covariant weight-one polar multiplet. This is a new type of supergravity compensator, although it is related to the  $q^+$ -hypermultiplet compensator which emerges within the harmonic superspace approach [13, 11] to 4D  $\mathcal{N} = 2$  supergravity, e.g., in the sense of [14]. In the present paper, we wish to study duality of the supergravity formulation given in [1] to (one of) the minimal  $(40 + 40)$  off-shell formulations for  $\mathcal{N} = 2$  Poincaré supergravity constructed in the 1980s. These formulations are obtained by coupling the minimal field representation with  $32 + 32$  off-shell degrees of freedom [15] (that is the Weyl multiplet [16, 17, 18] coupled to an Abelian vector multiplet, the latter being the first superconformal compensator) to various off-shell versions for the second compensator with  $(8 + 8)$  degrees of freedom. These include: (i) the “standard” minimal realization with a nonlinear multiplet [19, 16]; (ii) the alternative formulation involving an off-shell hypermultiplet with intrinsic central charge [20]; (iii) the new minimal realization with an improved tensor multiplet [21]. Superspace realizations for these supergravity formulations have been studied, e.g., in [22, 5, 11]. Our analysis will be specifically concerned with the duality between the third minimal Poincaré supergravity [21] and the supergravity formulation given in [1]. The point is that the former is known to be analogous to the new minimal  $\mathcal{N} = 1$  Poincaré supergravity [23]. We are going to demonstrate below that the projective-superspace formulation [1] is analogous to the old minimal  $\mathcal{N} = 1$  Poincaré supergravity [24, 25].<sup>3</sup>

This paper is organized as follows. In section 2, we elaborate upon the projective-superspace description of Abelian vector multiplets in  $\mathcal{N} = 2$  conformal supergravity, and present an alternative superspace representation for locally supersymmetric chiral actions. In section 3, we start by describing the superspace realization for the improved vector multiplet (both massless and massive) coupled to conformal supergravity. We then present new off-shell matter couplings within the third minimal Poincaré supergravity [21]. And finally, the duality of such supergravity-matter systems to those presented in [1] is explicitly proved. In the appendix we provide four equivalent forms for the free

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<sup>2</sup>We follow the terminology introduced in [12] in the rigid supersymmetric case.

<sup>3</sup>The “standard” minimal formulation for  $\mathcal{N} = 2$  Poincaré supergravity [19, 16] is analogous to the  $\mathcal{N} = 1$  non-minimal supergravity [26, 27], and thus it seems to be hardly useful for practical (e.g., supergraph) calculations. As to the alternative formulation for  $\mathcal{N} = 2$  supergravity [20], its best superspace description appears to be achieved within the harmonic superspace approach, as worked out in [28] and reviewed in [11].

$\mathcal{N} = 2$  vector multiplet action in conformal supergravity. Our two-component notation and conventions follow [29], and these are almost identical to those adopted in [30].

## 2 Vector multiplets in conformal supergravity

In this section, we elaborate upon the projective-superspace description of Abelian vector multiplets in conformal supergravity, building on [1, 31], and also propose an alternative superspace realization for  $\mathcal{N} = 2$  locally supersymmetric chiral actions. Following the supergravity conventions adopted [1], an Abelian vector multiplet is described by its field strength  $W(z)$  which is covariantly chiral,

$$\bar{\mathcal{D}}_{\dot{i}}^{\dot{\alpha}} W = 0 , \quad (2.1)$$

and obeys the Bianchi identity

$$\Sigma^{ij} := \frac{1}{4} \left( \mathcal{D}^{\gamma(i} \mathcal{D}_{\gamma}^{j)} + 4S^{ij} \right) W = \frac{1}{4} \left( \bar{\mathcal{D}}_{\dot{\gamma}}^{(i} \bar{\mathcal{D}}^{j)\dot{\gamma}} + 4\bar{S}^{ij} \right) \bar{W} =: \bar{\Sigma}^{ij} . \quad (2.2)$$

Under the infinitesimal super-Weyl transformation,  $W$  varies as

$$\delta_{\sigma} W = \sigma W , \quad (2.3)$$

with  $\sigma$  an arbitrary covariantly chiral scalar. The super-Weyl transformation of  $\Sigma^{ij}$  is

$$\delta_{\sigma} \Sigma^{ij} = (\sigma + \bar{\sigma}) \Sigma^{ij} . \quad (2.4)$$

The vector multiplet can also be described in terms of a gauge prepotential<sup>4</sup> which we identify with a covariant real weight-zero tropical supermultiplet,  $V(u^+)$ . In the north chart of  $\mathbb{CP}^1$  parametrized by the complex coordinate  $\zeta = u^{+2}/u^{+1}$ , the prepotential is specified by the following properties:

$$\mathcal{D}_{\alpha}^{+} V = \bar{\mathcal{D}}_{\dot{\alpha}}^{+} V = 0 , \quad V(u^+) = V(\zeta) = \sum_{k=0}^{+\infty} \zeta^k V_k , \quad V_k = (-1)^k \bar{V}_{-k} . \quad (2.5)$$

The prepotential is defined modulo gauge transformations of the form:

$$\delta V = \lambda + \tilde{\lambda} , \quad (2.6)$$

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<sup>4</sup>In the harmonic superspace approach [10, 11], one uses a different gauge prepotential which is globally defined on  $S^2 = \mathbb{CP}^1$ . The explicit relationship between the harmonic and the projective superspace formulations is spelled out in [14] in the rigid supersymmetric case.

with the gauge parameter  $\lambda(u^+)$  being a covariant weight-zero arctic multiplet, and  $\tilde{\lambda}$  its smile-conjugate (see, e.g., [1] for the definition of the smile-conjugation),

$$\mathcal{D}_\alpha^+ \lambda = \bar{\mathcal{D}}_\alpha^+ \lambda = 0, \quad \lambda(u^+) = \lambda(\zeta) = \sum_{k=0}^{+\infty} \zeta^k \lambda_k, \quad (2.7a)$$

$$\mathcal{D}_\alpha^+ \tilde{\lambda} = \bar{\mathcal{D}}_\alpha^+ \tilde{\lambda} = 0, \quad \tilde{\lambda}(u^+) = \tilde{\lambda}(\zeta) = \sum_{k=0}^{+\infty} (-1)^k \zeta^{-k} \bar{\lambda}_k. \quad (2.7b)$$

It turns out that the field strength  $W$  and its conjugate  $\bar{W}$  are expressed in terms of the prepotential  $V$  as follows [31]:

$$W = -\frac{1}{8\pi} \oint \frac{(u^+ du^+)}{(u^+ u^-)^2} \left( (\bar{\mathcal{D}}^-)^2 + 4\tilde{S}^{--} \right) V(u^+), \quad (2.8a)$$

$$\bar{W} = -\frac{1}{8\pi} \oint \frac{(u^+ du^+)}{(u^+ u^-)^2} \left( (\mathcal{D}^-)^2 + 4S^{--} \right) V(u^+), \quad (2.8b)$$

where the contour integral is carried out around the origin,  $\bar{\mathcal{D}}_\alpha^- = u_i^- \bar{\mathcal{D}}_\alpha^i$  and  $\mathcal{D}_\alpha^- = u_i^- \mathcal{D}_\alpha^i$ ,  $\tilde{S}^{\pm\pm} = u_i^\pm u_j^\pm \tilde{S}^{ij}$  and  $S^{\pm\pm} = u_i^\pm u_j^\pm S^{ij}$ . Here we have introduced an additional complex two-vector,  $u_i^-$ , which is only subject to the condition  $(u^+ u^-) := u^{+i} u_i^- \neq 0$ , and is otherwise completely arbitrary. The right-hand sides of (2.8a) and (2.8b) can be seen to be invariant under arbitrary projective transformations of the form:

$$(u_i^-, u_i^+) \rightarrow (u_i^-, u_i^+) R, \quad R = \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \in \text{GL}(2, \mathbb{C}). \quad (2.9)$$

The representations (2.8a) and (2.8b) generalize similar results in the 5D  $\mathcal{N} = 1$  flat [32] and Anti-de Sitter [33] superspaces.

Using the fact that  $V(u^+)$  is a covariant projective supermultiplet of weight zero, in particular  $\mathcal{D}_\alpha^+ V = \bar{\mathcal{D}}_\alpha^+ V = 0$ , one can show that the right-hand side of (2.8a) is covariantly chiral [31]. The field strength  $W$ , eq. (2.8a), turns out to be invariant under the gauge transformations (2.6) [31].

In accordance with the general results on the super-Weyl transformation laws of covariant projective multiplets [1], the gauge prepotential must be inert under the super-Weyl transformations,

$$\delta_\sigma V = 0. \quad (2.10)$$

It can be demonstrated that this transformation law implies the super-Weyl transformation of  $W$ , eq. (2.3).

Let  $\mathbf{V}(u^+)$  be the tropical prepotential for the vector multiplet given by the field strengths  $\mathbf{W}$  and  $\bar{\mathbf{W}}$  appearing in the supersymmetric action (1.2). The dynamics of this vector multiplet can be described by the Lagrangian [1]

$$\mathcal{L}_{\text{vector}}^{++} = -\frac{1}{2} \mathbf{V} \Sigma^{++} . \quad (2.11)$$

We wish to express the corresponding action,  $S_{\text{vector}}$ , in terms of the prepotential.

Making use of eq. (2.8a) gives

$$\begin{aligned} \Sigma^{++}(u^+) &= \frac{1}{4} \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \mathbf{W} \\ &= -\frac{1}{32\pi} \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \oint \frac{(\hat{u}^+ d\hat{u}^+)}{(\hat{u}^+ u^+)^2} \left( (\bar{\mathcal{D}}^-)^2 + 4\tilde{S}^{--} \right) \mathbf{V}(\hat{u}^+) . \end{aligned} \quad (2.12)$$

Here the expression in the second line does not depend on  $u_i^-$ , and the freedom to choose  $u_i^-$  can be used to set  $u_i^- = u_i^+$ . This leads to the following representation

$$\begin{aligned} \Sigma^{++}(u^+) &= -\frac{1}{32\pi} \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \left( (\bar{\mathcal{D}}^+)^2 + 4\tilde{S}^{++} \right) \oint \frac{(\hat{u}^+ d\hat{u}^+)}{(\hat{u}^+ u^+)^2} \mathbf{V}(\hat{u}^+) \\ &= -\frac{1}{32\pi} \left( (\bar{\mathcal{D}}^+)^2 + 4\tilde{S}^{++} \right) \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \oint \frac{(\hat{u}^+ d\hat{u}^+)}{(\hat{u}^+ u^+)^2} \mathbf{V}(\hat{u}^+) , \end{aligned} \quad (2.13)$$

which makes manifest the fact that  $\Sigma^{++}$  is a covariant projective multiplet. Plugging the expression obtained into the action  $S_{\text{vector}}$  and then integrating by parts gives

$$\begin{aligned} S_{\text{vector}} &= \frac{1}{2\pi} \oint (u^+ du^+) \int d^4x d^4\theta d^4\bar{\theta} E \frac{\mathbf{W} \bar{\mathbf{W}} \mathcal{L}_{\text{vector}}^{++}}{(\Sigma^{++})^2} \\ &= \frac{1}{2} \frac{1}{(2\pi)^2} \oint (u_1^+ du_1^+) \oint (u_2^+ du_2^+) \int d^4x d^4\theta d^4\bar{\theta} E \frac{\mathbf{V}(u_1^+) \mathbf{V}(u_2^+)}{(u_1^+ u_2^+)^2} . \end{aligned} \quad (2.14)$$

This result is a curved-superspace generalization of the rigid supersymmetric action for the vector multiplet in projective superspace [34]. The fact that here we have dealt with a particular vector multiplet, is not actually relevant. In the appendix, we generalize (2.14) to the case of an arbitrary vector multiplet.

The description of vector multiplets in terms of their projective prepotentials may look somewhat exotic. Having this in mind, we would like to demonstrate its equivalence to the standard formulation, in which the vector multiplet action is given as an integral over the chiral subspace, originally presented in [35] in the case of rigid supersymmetry and then extended to supergravity, e.g., in [36]. Since the curved-superspace considerations require the use of a chiral density (see [36] and references therein), which makes the

analysis somewhat lengthy and technical, we restrict ourselves to the flat case. Using the flat-superspace representation

$$\Sigma^{++} = \frac{1}{4}(D^+)^2 \mathbf{W} = \frac{1}{4}(\bar{D}^+)^2 \bar{\mathbf{W}} , \quad (2.15)$$

$S_{\text{vector}}$  can be transformed as follows:

$$\begin{aligned} S_{\text{vector}} &= -\frac{1}{4\pi} \oint (u^+ du^+) \int d^4x d^4\theta d^4\bar{\theta} \frac{\mathbf{W} \bar{\mathbf{W}}}{\Sigma^{++}} \mathbf{V} \\ &= -\frac{1}{64\pi} \oint (u^+ du^+) \int d^4x d^4\theta \frac{(\bar{D}^-)^2 (\bar{D}^+)^2}{(u^+ u^-)^2} \frac{\mathbf{W} \bar{\mathbf{W}}}{\Sigma^{++}} \mathbf{V} \\ &= -\frac{1}{16\pi} \oint \frac{(u^+ du^+)}{(u^+ u^-)^2} \int d^4x d^4\theta \mathbf{W} (\bar{D}^-)^2 \mathbf{V} = \frac{1}{2} \int d^4x d^4\theta \mathbf{W}^2 . \end{aligned} \quad (2.16)$$

In the last line, we have used the flat-superspace version of eq. (2.8a).

As a natural generalization, consider a system of  $n + 1$  Abelian vector multiplets described by covariantly chiral field strengths  $W_I$ , where  $I = 0, 1, \dots, n$ , and  $W_0 = \mathbf{W}$ . Their dynamics can be described by the Lagrangian [1]

$$\mathcal{L}^{++} = -\frac{1}{4} \mathbf{V} \left\{ \left( (\mathcal{D}^+)^2 + 4S^{++} \right) F(W_I) + \left( (\bar{\mathcal{D}}^+)^2 + 4\tilde{S}^{++} \right) \bar{F}(\bar{W}_I) \right\} , \quad (2.17)$$

with  $F(W_I)$  a holomorphic homogeneous function of degree one,  $F(cW_I) = cF(W_I)$ . The construction given admits an obvious extension to the non-Abelian case.

In the *flat-superspace* limit, the action generated by (2.17) can be represented in the following different but equivalent forms:

$$S = \frac{1}{2\pi} \oint (u^+ du^+) \int d^4x d^4\theta d^4\bar{\theta} \frac{\mathbf{W} \bar{\mathbf{W}} \mathcal{L}^{++}}{(\Sigma^{++})^2} = \int d^4x d^4\theta \mathbf{W}^2 F\left(\frac{W_I}{\mathbf{W}}\right) + \text{c.c.} \quad (2.18)$$

It should be pointed out that the representation (2.17), which describes effective vector multiplet models in supergravity, is a natural generalization of that given in [37] in the rigid supersymmetric case using harmonic superspace techniques.

The above consideration leads to a new representation for chiral actions in  $\mathcal{N} = 2$  supergravity that avoids any use of the chiral density (as mentioned earlier, the latter requires some care to be explicitly constructed [36]). Let  $\mathcal{L}_c(z)$  be a covariantly chiral scalar superfield,  $\bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{L}_c = 0$ , with the super-Weyl transformation

$$\delta_{\sigma} \mathcal{L}_c = 2\sigma \mathcal{L}_c . \quad (2.19)$$

For the chiral action  $S_c$  associated with  $\mathcal{L}_c$ , we have

$$S_c = \int d^4x d^4\theta \mathcal{E} \mathcal{L}_c + \text{c.c.} = \frac{1}{2\pi} \oint (u^+ du^+) \int d^4x d^4\theta d^4\bar{\theta} E \frac{\mathbf{W} \bar{\mathbf{W}} \mathcal{L}_c^{++}}{(\Sigma^{++})^2},$$

$$\mathcal{L}_c^{++} = -\frac{1}{4} \mathbf{V} \left\{ \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \frac{\mathcal{L}_c}{\mathbf{W}} + \left( (\bar{\mathcal{D}}^+)^2 + 4\tilde{S}^{++} \right) \frac{\bar{\mathcal{L}}_c}{\bar{\mathbf{W}}} \right\}. \quad (2.20)$$

If  $\mathcal{L}_c$  is independent of the vector multiplet described by  $\mathbf{V}$ , then one can show, using the representations (2.8a) and (2.8b), that  $S_c$  does not change under an arbitrary variation of the prepotential  $\mathbf{V}$ ,

$$\frac{\delta}{\delta \mathbf{V}} \mathcal{L}_c = 0 \quad \implies \quad \frac{\delta}{\delta \mathbf{V}} S_c = 0. \quad (2.21)$$

The derivation of this result requires transformations similar to those described in the Appendix.

As an example of chiral models, consider a higher-derivative Lagrangian of the form:

$$\mathcal{L}_c = \frac{(W^{\alpha\beta} W_{\alpha\beta})^n}{(\mathbf{W})^{2n-2}}, \quad (2.22)$$

with  $W_{\alpha\beta}$  the  $\mathcal{N} = 2$  super-Weyl tensor.

### 3 Dual formulations for matter-coupled supergravity

We are prepared for the analysis of supergravity-matter systems and their dualities.<sup>5</sup>

#### 3.1 Massless and massive improved tensor multiplets

The improved  $\mathcal{N} = 2$  tensor multiplet<sup>6</sup> [21, 40, 7, 41] occurs as a conformal compensator in one of the off-shell formulations for 4D  $\mathcal{N} = 2$  Poincaré supergravity which was developed in [21] using the  $\mathcal{N} = 2$  superconformal tensor calculus. Here we start by presenting a curved superspace realization for the improved tensor multiplet, building on the rigid projective superspace formulation for this multiplet given in [7].

The  $\mathcal{N} = 2$  tensor multiplet<sup>7</sup> is described by a covariant real  $O(2)$  multiplet  $G^{++}$ ,

$$G^{++}(u^+) = G^{ij} u_i^+ u_j^+, \quad \overline{G^{ij}} = G_{ij}, \quad \mathcal{D}_\alpha^{(i} G^{jk)} = \bar{\mathcal{D}}_{\dot{\alpha}}^{(i} G^{jk)} = 0. \quad (3.1)$$

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<sup>5</sup>Examples of duality transformations for rigid projective supermultiplets were considered, e.g., in [12].

<sup>6</sup>The improved  $\mathcal{N} = 1$  tensor multiplet was introduced by de Wit and Roček [38]. It is a unique superconformal model in the family of  $\mathcal{N} = 1$  tensor multiplet models discovered by Siegel [39].

<sup>7</sup>In rigid  $\mathcal{N} = 2$  supersymmetry, the off-shell tensor multiplet was first introduced by Wess [42], and its projective superspace realization was given in [7].



The Lagrangian for the improved tensor multiplet is

$$\mathcal{L}_{\text{impr.}-\text{tensor}}^{++} = -G^{++} \ln \frac{G^{++}}{i\tilde{\Upsilon}^+ \Upsilon^+} , \quad (3.2)$$

where  $\Upsilon^+(u^+)$  is a covariant weight-one *arctic* multiplet, and  $\tilde{\Upsilon}^+(u^+)$  its smile-conjugated *antarctic* superfield. The action can be seen to be independent of  $\Upsilon^+$  and  $\tilde{\Upsilon}^+$ . Indeed, one can show that

$$\oint (u^+ du^+) \int d^4x d^4\theta d^4\bar{\theta} E \frac{W\bar{W}}{(\Sigma^{++})^2} G^{++} \lambda = 0 , \quad (3.3)$$

for an arbitrary covariant weight-zero *arctic* multiplet  $\lambda(u^+)$ . Therefore, the action generated by (3.2) is invariant under gauge transformations  $\Upsilon^+ \rightarrow e^\lambda \Upsilon^+$ , and thus  $\Upsilon^+$  can be gauged away. In other words,  $\Upsilon^+$  is a purely degree of freedom. Both  $G^{++}$  and  $\Upsilon^+$  are required to possess non-vanishing expectation values.

The action generated by (3.2) is manifestly invariant under the super-Weyl transformations

$$\delta_\sigma G^{++} = (\sigma + \bar{\sigma}) G^{++} , \quad \delta_\sigma \Upsilon^+ = \frac{1}{2}(\sigma + \bar{\sigma}) \Upsilon^+ . \quad (3.4)$$

These super-Weyl transformation laws of  $G^{++}$  and  $\Upsilon^+$  are determined by their off-shell structure [1]. In the flat superspace limit, the Lagrangian (3.2) provides a *manifestly superconformal formulation* for the improved tensor multiplet within the superconformal formalism given in [6].

One can consider a coupling of  $G^{++}$  to an Abelian vector multiplet generated by the Lagrangian

$$\mathcal{L}^{++} = -\frac{1}{2} V \Sigma^{++} - G^{++} \ln \frac{G^{++}}{i\tilde{\Upsilon}^+ e^{mV} \Upsilon^+} , \quad (3.5)$$

with  $m$  a real parameter. The corresponding action is invariant under the gauge transformations (2.6). This model describes a massive improved tensor multiplet. In the rigid supersymmetric case, it was introduced in [40] in terms of  $\mathcal{N} = 1$  superfields (as a model for  $\mathcal{N} = 2$  supersymmetric QED), then in [43] in terms of ordinary  $\mathcal{N} = 2$  superfields, and later its description in  $\mathcal{N} = 2$  projective superspace was given [44].

Massive two-forms naturally appear in four-dimensional  $\mathcal{N} = 2$  supergravity theories obtained from (or related to) compactifications of type II string theory on Calabi-Yau threefolds in the presence of both electric and magnetic fluxes [45, 46]. This led to renewed interest in  $\mathcal{N} = 1$  and  $\mathcal{N} = 2$  rigid massive tensor multiplets<sup>8</sup> [47, 48, 49] some

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<sup>8</sup>Models for massive  $\mathcal{N} = 1$  tensor multiplet were proposed for the first time in [39].

time ago. Here we wish to give an alternative formulation, as compared with (3.2), for the massive improved  $\mathcal{N} = 2$  tensor multiplet in conformal supergravity building on the rigid supersymmetric construction of [49].

Let us introduce a covariantly chiral prepotential  $\Psi$  for the tensor multiplet (see, e.g., [50] and references therein)

$$G^{++}(u^+) = \frac{1}{8} \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \Psi + \frac{1}{8} \left( (\bar{\mathcal{D}}^+)^2 + 4\tilde{S}^{++} \right) \bar{\Psi} , \quad \bar{\mathcal{D}}_{\dot{\alpha}}^i \Psi = 0 \quad (3.6)$$

The prepotential is defined modulo gauge transformations of the form:

$$\delta \Psi = i \Lambda , \quad \bar{\mathcal{D}}_{\dot{\alpha}}^i \Lambda = 0 , \quad \left( \mathcal{D}^{\gamma(i} \mathcal{D}^{j)} + 4S^{ij} \right) \Lambda = \left( \bar{\mathcal{D}}_{\dot{\gamma}}^{(i} \bar{\mathcal{D}}^{j)} + 4\bar{S}^{ij} \right) \bar{\Lambda} . \quad (3.7)$$

The super-Weyl transformation law of  $\Psi$  should be

$$\delta_{\sigma} \Psi = \sigma \Psi \quad (3.8)$$

in order for  $G^{++}$  to transform as in eq. (3.4).

To describe a massive improved tensor multiplet, one can choose the following Lagrangian:

$$\begin{aligned} \mathcal{L}^{++} = & -G^{++} \ln \frac{G^{++}}{i\tilde{\Upsilon}^+ \Upsilon^+} \\ & + \frac{1}{16} \mathbf{V} \left\{ \mu(\mu + ie) \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \frac{\Psi^2}{\mathbf{W}} + \text{smile-conjugate} \right\} , \end{aligned} \quad (3.9)$$

with  $\mu$  and  $e$  constant mass parameters. Here the mass terms include both “electric” and “magnetic” contributions. The action generated by this Lagrangian is obviously super-Weyl invariant. The mass parameter in (3.9) is complex, and can be interpreted as a vacuum expectation value for vector multiplets<sup>9</sup>

$$\mu(\mu + ie) \int d^4x d^4\theta \mathcal{E} \Psi^2 \quad \longleftarrow \quad \int d^4x d^4\theta \mathcal{E} H(W) \Psi^2 , \quad (3.10)$$

with  $H(W)$  a holomorphic homogeneous function of degree zero, with its variables  $W$ ’s being the field strengths of Abelian vector multiplets.

The rigid supersymmetric versions of (3.5) and (3.9) are dually equivalent provided the mass parameters are related as [49]

$$m^2 = \mu^2 + e^2 . \quad (3.11)$$

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<sup>9</sup>This interpretation is inspired by [51].

It turns out that this duality extends to supergravity. Indeed, let us consider the auxiliary first-order Lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{aux}}^{++} = & -U^{++} \ln \left( \frac{U^{++}}{i\tilde{\Upsilon}^+ \Upsilon^+} - 1 \right) \\ & + mV \left\{ U^{++} - \frac{1}{8} \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \Psi - \frac{1}{8} \left( (\bar{\mathcal{D}}^+)^2 + 4\tilde{S}^{++} \right) \bar{\Psi} \right\} \\ & + \frac{1}{16} \mathbf{V} \left\{ \mu(\mu + ie) \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \frac{\Psi^2}{\mathbf{W}} + \text{smile-conjugate} \right\} .\end{aligned}\quad (3.12)$$

Here  $U^{++}$  and  $V$  are covariant real weight-two and weight-zero tropical multiplets, respectively, and  $\Psi$  a covariantly chiral scalar. The corresponding action,  $S_{\text{aux}}$ , is invariant under the gauge transformation (2.6) that also acts on  $\Upsilon^+$  as  $\delta\Upsilon^+ = -m\lambda\Upsilon^+$ , and hence  $\Upsilon^+$  is a purely gauge degree of freedom. Varying<sup>10</sup>  $S_{\text{aux}}$  with respect to  $V$  leads to  $U^{++} = G^{++}$ , with  $G^{++}$  given by eq. (3.6), and then we arrive at the theory with Lagrangian (3.9). On the other hand, varying  $U^{++}$  and  $\Psi$  can be shown to lead to the following model:

$$\mathcal{L}_{\text{dual}}^{++} = -\frac{1}{16} \mathbf{V} \left\{ \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \frac{W^2}{\mathbf{W}} + \left( (\bar{\mathcal{D}}^+)^2 + 4\tilde{S}^{++} \right) \frac{\bar{W}^2}{\bar{\mathbf{W}}} \right\} + i\tilde{\Upsilon}^+ e^{mV} \Upsilon^+ , \quad (3.13)$$

where  $W$  is the field strength, eq. (2.8a), associated with the gauge prepotential  $V$ . One can further demonstrate that the theories (3.5) and (3.5), in complete analogy with the consideration in subsection 3.3 below. This completes the proof.

In order to evaluate the variational derivatives of  $S_{\text{aux}}$  with respect  $\Psi$ , a few comments are actually in order. First of all, using the representation (2.8a) for the field strengths  $\mathbf{W}$ , in conjunction with integration by parts, one can transform the linear in  $\Psi$  term in  $S_{\text{aux}}$  as follows:

$$\begin{aligned}& -\frac{m}{16\pi} \int d^4x d^4\theta d^4\bar{\theta} E \oint (u^+ du^+) \frac{\mathbf{W}\bar{\mathbf{W}}}{(\Sigma^{++})^2} V \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \Psi \\ & = \frac{m}{8\pi^2} \int d^4x d^4\theta d^4\bar{\theta} E \oint (u^+ du^+) \oint (\hat{u}^+ d\hat{u}^+) \frac{V(u^+) \mathbf{V}(\hat{u}^+)}{(\hat{u}^+ u^+)^2} \frac{\Psi}{\mathbf{W}} .\end{aligned}$$

Relabelling here  $u^+ \leftrightarrow \hat{u}^+$ , then making use of the representation (2.8a) for the field strength  $W$  associated with  $V$ , and also integrating by parts, the latter expression can be brought to the form:

$$-\frac{m}{16\pi} \int d^4x d^4\theta d^4\bar{\theta} E \oint (u^+ du^+) \frac{\mathbf{W}\bar{\mathbf{W}}}{(\Sigma^{++})^2} \mathbf{V} \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \frac{\Psi W}{\mathbf{W}} .$$

---

<sup>10</sup>The equations of motion for  $\Upsilon^+$  and  $\tilde{\Upsilon}^+$  only force  $U^{++}$  to be a tensor multiplet. The equation of motion for  $V$  requires  $\Psi$  to be the prepotential of this tensor multiplet.

Now, recalling the two equivalent representations (2.20) for chiral actions, the  $\Psi$ -dependent terms in  $S_{\text{aux}}$  become

$$-\frac{1}{4} \int d^4x d^4\theta \mathcal{E} \left\{ \mu(\mu + ie)\Psi^2 - 2m\Psi W \right\} ,$$

and this functional is trivial to vary with respect to  $\Psi$ .

### 3.2 Supergravity-matter systems with tensor compensator

We now turn to supergravity-matter systems. To start with, we choose the following compensating multiplets: (i) the vector multiplet described by its covariant real weight-zero *tropical* prepotential  $\mathbf{V}(u^+)$ , with  $\mathbf{W}$  the corresponding gauge-invariant covariantly chiral field strength; and (ii) the tensor multiplet  $\mathbf{G}^{++}(u^+)$ . As matter fields, we choose a set of covariant weight-zero arctic multiplets  $\Upsilon^I(u^+)$  and their smile-conjugates  $\tilde{\Upsilon}^{\bar{I}}(u^+)$  which take their values in a Kähler manifold, with  $K(v^I, \bar{v}^{\bar{J}})$  the corresponding Kähler potential. The supergravity-matter Lagrangian is

$$\mathcal{L}_{\text{SM-tensor}}^{++} = \frac{1}{2} \mathbf{V} \Sigma^{++} + \mathbf{G}^{++} \left( \ln \frac{\mathbf{G}^{++}}{i\tilde{\Upsilon}^+ e^m \mathbf{V} \Upsilon^+} - K(\Upsilon, \tilde{\Upsilon}) \right) , \quad (3.14)$$

with  $m$  a cosmological constant. It should be pointed out that the vector and the tensor multiplet kinetic terms appear here with wrong signs, as compared with (2.11) and (3.2). Since  $K(v^I, \bar{v}^{\bar{J}})$  is essentially arbitrary, and the covariant polar multiplets were discovered in [1], the theory introduced describes more general matter couplings than previously constructed within the third minimal formulation for  $\mathcal{N} = 2$  Poincaré supergravity [21].

The action generated by (3.14) is invariant under the gauge transformations of the compensating vector multiplet, eq. (2.6). It is also super-Weyl invariant, since the covariant weight-zero projective multiplets are invariant under such transformations [1],

$$\delta_\sigma \Upsilon^I = 0 . \quad (3.15)$$

In addition, the action possesses the Kähler invariance

$$K(\Upsilon, \tilde{\Upsilon}) \rightarrow K(\Upsilon, \tilde{\Upsilon}) + \Lambda(\Upsilon) + \bar{\Lambda}(\tilde{\Upsilon}) , \quad (3.16)$$

with  $\Lambda$  an arbitrary holomorphic function.

It is interesting to note that the equation of motion for  $\mathbf{V}$  is

$$\Sigma^{++} + m\mathbf{G}^{++} = 0 . \quad (3.17)$$

Then, the dynamics of the matter sector is generated by the Lagrangian

$$\mathcal{L}^{++} \propto \Sigma^{++} K(\Upsilon, \tilde{\Upsilon}) . \quad (3.18)$$

A similar model has been introduced in [3] in the case of 5D  $\mathcal{N} = 1$  supergravity.

### 3.3 Supergravity-matter systems with polar compensator

Let us derive a dual formulation for the theory (3.14). Instead of (3.14), we consider the following first-order Lagrangian:

$$\mathcal{L}_{\text{first-order}}^{++} = \frac{1}{2} \mathbf{V} \Sigma^{++} + U^{++} \left( \ln \frac{U^{++}}{\mathbf{i} \tilde{\Upsilon}^+ e^m \mathbf{V} \Upsilon^+} - 1 - K(\Upsilon, \tilde{\Upsilon}) \right) . \quad (3.19)$$

Here  $U^{++}$  is a covariant real weight-two *tropical* multiplet, and  $\Upsilon^+$  a covariant weight-one arctic multiplet. Unlike the purely gauge superfield  $\Upsilon^+$  in the original model (3.14), the  $\Upsilon^+$  is now a non-trivial dynamical variable. The first-order model introduced respects all the symmetries of the original theory (3.14), albeit in a modified form. The gauge invariance (2.6) turns into

$$\delta \mathbf{V} = \lambda + \tilde{\lambda} , \quad \delta \Upsilon^+ = -m \lambda \Upsilon^+ . \quad (3.20)$$

The Kähler transformation (3.16) becomes

$$K(\Upsilon, \tilde{\Upsilon}) \rightarrow K(\Upsilon, \tilde{\Upsilon}) + \Lambda(\Upsilon) + \bar{\Lambda}(\tilde{\Upsilon}) , \quad \Upsilon^+ \rightarrow e^{-\Lambda(\Upsilon)} \Upsilon^+ . \quad (3.21)$$

Finally, if the super-Weyl transformation of  $U^{++}$  is chosen to be

$$\delta_\sigma U^{++} = (\sigma + \bar{\sigma}) U^{++} , \quad (3.22)$$

then the action  $S_{\text{first-order}}$  associated with (3.19) is super-Weyl invariant.

Varying  $S_{\text{first-order}}$  with respect to  $\Upsilon^+$  and  $\tilde{\Upsilon}^+$  constrains  $U^{++}$  to be an  $O(2)$ -multiplet,  $U^{++} = G^{++}$ , and then  $S_{\text{first-order}}$  reduces to the action generated by (3.14). Therefore, the dynamical systems (3.14) and (3.19) are equivalent. On the other hand, varying  $S_{\text{first-order}}$  with respect to  $U^{++}$  leads to the following theory:

$$\mathcal{L}_{\text{SM-polar}}^{++} = \frac{1}{2} \mathbf{V} \Sigma^{++} - \mathbf{i} \tilde{\Upsilon}^+ e^m \mathbf{V}^{-K(\Upsilon, \tilde{\Upsilon})} \Upsilon^+ . \quad (3.23)$$

This is exactly the supergravity-matter system introduced in [1]. Its hypermultiplet sector is a curved-space version of the general 4D  $\mathcal{N} = 2$  superconformal sigma-model for polar multiplets proposed in [6] (building on the 5D  $\mathcal{N} = 1$  construction of [32]).

It is instructive to compare (3.14) and (3.23) to the well-known descriptions of matter couplings in the new minimal and the old minimal formulations for  $\mathcal{N} = 1$  supergravity (see, e.g., [29] for a review). The action for the matter-coupled new minimal supergravity is as follows:

$$S = \int d^4x d^2\theta d^2\bar{\theta} E \left\{ 3G \ln \frac{G}{\psi\bar{\psi}} + G K(\phi, \bar{\phi}) \right\} . \quad (3.24)$$

Here the compensator  $G$  is covariantly real linear, and  $\psi$  is a covariantly chiral scalar which is a pure gauge degree of freedom. The supersymmetric matter is described by covariantly chiral scalars  $\phi^I$  which are inert under the super-Weyl transformations. The action is super-Weyl invariant, and also possesses the Kähler invariance

$$K(\phi, \bar{\phi}) \rightarrow K(\phi, \bar{\phi}) + \Lambda(\phi) + \bar{\Lambda}(\bar{\phi}) , \quad (3.25)$$

with  $\Lambda(\phi)$  a holomorphic function. The action for the matter-coupled old minimal supergravity is

$$S = -3 \int d^4x d^2\theta d^2\bar{\theta} E \bar{\Psi} \Psi \exp\left(-\frac{1}{3} K(\phi, \bar{\phi})\right) , \quad (3.26)$$

where the compensator  $\Psi$  is covariantly chiral. The two theories (3.24) and (3.26) are dually equivalent. Clearly, the  $\mathcal{N} = 2$  supergravity theories (3.14) and (3.23) are generalizations of (3.24) and (3.26), respectively.

In principle, the supergravity-matter systems (3.14) and (3.23) can be used to generate arbitrary quaternion-Kähler geometries allowed in supergravity. In this respect, the results of [52] should also be relevant.

Ten years ago, it was advocated in [14] that the harmonic [10, 11] and projective [7, 8] superspace approaches provide complementary descriptions of rigid  $\mathcal{N} = 2$  supersymmetric theories. This also appears to hold at the level of  $\mathcal{N} = 2$  supergravity. The strong features of the projective-superspace formulation proposed in [1] are: (i) its geometric character; (ii) reasonably short off-shell hypermultiplets. The strongest point of the harmonic-superspace approach to  $\mathcal{N} = 2$  supergravity [13, 11] is a remarkably simple structure of the supergravity prepotentials. In particular, the latter approach provides a natural origin for the real scalar prepotential that generates the  $\mathcal{N} = 2$  supercurrent [44, 28]. It would be important to understand how such a prepotential originates within the projective-superspace scheme.

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## A Equivalent forms for the vector multiplet action

Here we list four equivalent forms for the free vector multiplet action:

$$S_{\text{VM}} = \frac{1}{2} \int d^4x d^4\theta \mathcal{E} W^2 = \frac{1}{4} \int d^4x d^4\theta \mathcal{E} W^2 + \text{c.c.} \quad (\text{A.1a})$$

$$= \frac{1}{2\pi} \oint (u^+ du^+) \int d^4x d^4\theta d^4\bar{\theta} E \frac{\mathbf{W} \bar{\mathbf{W}}}{(\Sigma^{++})^2} \mathcal{L}_1^{++} \quad (\text{A.1b})$$

$$= \frac{1}{2\pi} \oint (u^+ du^+) \int d^4x d^4\theta d^4\bar{\theta} E \frac{\mathbf{W} \bar{\mathbf{W}}}{(\Sigma^{++})^2} \mathcal{L}_2^{++} \quad (\text{A.1c})$$

$$= \frac{1}{2} \frac{1}{(2\pi)^2} \oint (u_1^+ du_1^+) \oint (u_2^+ du_2^+) \int d^4x d^4\theta d^4\bar{\theta} E \frac{V(u_1^+) V(u_2^+)}{(u_1^+ u_2^+)^2}, \quad (\text{A.1d})$$

where

$$\mathcal{L}_1^{++} = -\frac{1}{2} V \Sigma^{++}, \quad (\text{A.2a})$$

$$\mathcal{L}_2^{++} = -\frac{1}{16} \mathbf{V} \left\{ \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \frac{W^2}{\mathbf{W}} + \left( (\bar{\mathcal{D}}^+)^2 + 4\tilde{S}^{++} \right) \frac{\bar{W}^2}{\bar{\mathbf{W}}} \right\}. \quad (\text{A.2b})$$

Let us derive, for instance, (A.1d) from (A.1c). It is sufficient to consider the  $W^2$ -dependent part of (A.1c). Integrating by parts using the representation (2.8a) for the field strengths  $W$ , and integrating by parts once more, one can show that

$$\begin{aligned} & -\frac{1}{32\pi} \int d^4x d^4\theta d^4\bar{\theta} E \oint (u^+ du^+) \frac{\mathbf{W} \bar{\mathbf{W}}}{(\Sigma^{++})^2} \mathbf{V} \left( (\mathcal{D}^+)^2 + 4S^{++} \right) \frac{W^2}{\mathbf{W}} \\ & = \frac{1}{16\pi^2} \int d^4x d^4\theta d^4\bar{\theta} E \oint (u^+ du^+) \oint (\hat{u}^+ d\hat{u}^+) \frac{W}{\mathbf{W}} \frac{\mathbf{V}(u^+) V(\hat{u}^+)}{(\hat{u}^+ u^+)^2}. \end{aligned}$$

As a next step, one can re-label  $u^+ \leftrightarrow \hat{u}^+$  in the expression obtained, insert the unity resolved as  $\Sigma^{++}/\Sigma^{++}$ , and then integrate by parts, thus ending up with

$$-\frac{1}{8\pi} \int d^4x d^4\theta d^4\bar{\theta} E \oint (u^+ du^+) \frac{\bar{\mathbf{W}}}{\Sigma^{++}} V W.$$

It remains to make use, once more, of the representation (2.8a) for the field strengths  $W$ , and then integrate by part, in order to arrive at (A.1d).

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